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Bose-Einstein Condensation Controlled by (HOP+OLP) Trapping Potentials

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Abstract:-This work will focus on theoretical treatment of one-dimension harmonic oscillator trapping potential HOP applied along the x-axis together with an optical lattice trapping potential OLP applied along y-axis. These trapping potentialsare usually used in experiments that lead to produced Bose-Einstein condensation BEC in ultra-cold gases. This analysis will concentration mainly on the anisotropy parameter in HOPand q parameter in OLP, and the role that these factors play in term of values and shapes of the trapping potential, initial wave function, and final wave function. This specific study will give us the overall view of the region of confinement that the HOP and OLPhave employed. The study is extended to the chemical potential of the working fluid.

Keywords: *Laser cooled atom, BEC atom, Trapping, Quantum Oscillator*

I. INTRODUCTION

A periodic potential can be designed simplyby overlapping two counter-propagating laser beams [1]. The interference between the two laser beams forms an optical standing wave with period $\lambda_L/2$, which can trap the atoms. By interfering more laser beams, onecan obtain one, two, and three-dimensional (1D, 2D and 3D) periodic potentials [2-5]. The 1D lattice, formed by a pair of laser beams, generates a single standing wave interference pattern excellently an array of 2D disk-like trapping potentials [6,7]. Two orthogonal optical standing waves can create an array of 1D potential tube, in which the atoms can only move along the weakly confining axis of the potential tube, thus grasping 1D quantum performance, with the radial motion being completely frozen out for low-enough temperatures. Three orthogonal optical standing waves correspond to a 3D simple cubic crystal, in which each trapping site acts as a tightly confining harmonic oscillator potential. One important advantage of using optical fieldsto create a periodic trapping potential is that the geometry and depth of the potential are under acomplete control of the experimentalist. For example,the geometry of the trapping potentials can be alteredby interfering laser beams under a different angle, thus making even more complex lattice arrangements [8], such as Kagome lattices [9]. The deepness of such optical potentials can even be varied dynamically during an experimental sequence by simply increasing or decreasing the intensity of the laser light, thus turning experimental investigations of the time dynamics of essential phase transitions into a reality.As a simulator, an optical lattice offersremarkably clean access to a particular Hamiltonianand thereby assists as a model system for testing fundamental theoretical concepts, at times providing good examples of quantum many-body effects.Usually, ultra-cold neutral atoms are put in storage inmagnetic traps, in which only a small subset of the available atomic spin states, the so-called weak field-seeking states can be trapped. This restrictionis generally overcome by using optical dipole traps that rely on the interaction between an induced dipole moment in an atom and an external electric field. Such a field can, for example, be provided bythe oscillating electric light field from a laser, which induces an oscillating dipole moment in the atom while at the same time interacts with this dipole moment in order to create a trapping potential$V_{dip}(\mathbf{r})$ for the atoms [10]. In this study we will focus on the relations that connect the chemical potential and wave function with the anisotropy of the HOP, and the chemical potential with the parameter q of the OLP. The study is extended to analysis the distribution of the wave function through the working area. We have confirmed that the parameters in HOP, and OLP play a major part in the solution of the GPE and care should be taking into account if one like to bring his analysis to a satisfactory experiments result.

II. THEORY

The Gross-Pitaevskii Equation (GPE), is the result of a mean-fieldtheory that's most useful in treatinghighly dilute and ultra-cold BECs withnegligible many-body correlation effects.It has also been useful in dealing with physicalsituations in which a cloud of thermally excitedbosonic atoms or fermionic atoms accompany agaseous condensate. In these cases, the GPE's numericalsolution must be found in conjunction withthe solution of appropriate equations for the coexistingcloud, such as the Vlasov-Landau equations [11]. Under these conditions it is worth first to write down the GPE. If we consider the macroscopic dynamics of a BECloaded onto an elongated optical lattice, similar to thosecreated in a number of experiments (see, e.g., Refs.[12-14]). The condensate dynamics is described by thethree-dimensional GP equation,

$$i\hbar\frac{\partial\psi}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+V(x,y,z)+\Gamma|\psi|^2\right]\psi \quad (1)$$

Where $\psi(x,y,z,t)$ is the wave function of the cigar-shapedcondensate, (y,z) are the directions of strongtransverse confinement, and y is the direction of thelattice. The combined potential of the optical latticeand magnetic trap V(x, y,z) can be written as,

$$V(y,r^2)=E_0\sin^2\left(\frac{\pi y}{d}\right)+\frac{1}{2}m(\omega_x^2x^2+\omega_\perp^2r^2) \quad (2)$$

E_0 is the well depth of the opticallattice, d is the characteristic lattice constant, and $\omega_i$are trapping frequencies in the corresponding directions.The parameter $\Gamma=4\pi g\hbar^2a_s/m$ characterizes the s-wavescattering of atoms in the condensate which introducesan effective nonlinearity in the mean-field equation; it ispositive for repulsive interactions and negative for attractiveinteractions.Measuring time in units of $\omega_\perp^{-1}$, the spatial variablesin units of the transverse harmonic oscillator length,$a_0=(\hbar/m\omega_\perp)^{1/2}$, the wave function amplitude in unitsof $a_0^{-3/2}$, and the potential in units of $\hbar\omega_\perp$, we obtain thefollowing dimensionless GP equation:

$$i\frac{\partial\psi}{\partial t}=\left[-\frac{1}{2}\nabla^2+V(x,y)+G_{2d}|\psi|^2\right]\psi \quad (3)$$

Where,$t=T\omega_\perp$, and $G_{2d}=4\pi(a_s/a_0)$. The potential now takes the form:

$$V(x,y)=V_0\sin^2(qy)+\frac{1}{2}(AL)^2x^2 \quad (4)$$

Where, $V_0=E_0/(\hbar\omega_\perp)$,and $q=\pi a_0/d$. The ratio of the confinement for the magnetic trap$AL=\omega_x/\omega_\perp$varies from 10^{-1} to,$1/\sqrt{2}$, see for example [15-18], but this is not the case in this study.The Crank-Nicolson (CN) scheme is the slogger for the numerical integration of quantum wave equations. The method's major advantage is that it ensures unitarily for an arbitrary size of the time step dt. This lets the wave function advance by large steps—that is, larger than the inverse of the highest eigen value,$\omega_{MAX}=E_{max}/\hbar$. Only accuracy (at the second order) limits the time-step size, not stability or unitarily considerations. The CN method requires us to solve a matrix algebraic problem at each time step. This is a fairly expensive computational task.

To solve the GPE several properties should be assign that are:

a) A first property of the GPE is the time invariance. This means that the equation is time reversible: the equation is stable with respect to the time change of variable $t\to -t$.
b) A second property is known as invariance by phase shift or gauge invariance: if $\widetilde{\psi}$ is the solution to equation (3), then $\psi=\widetilde{\psi}e^{-\alpha t}$ is solution to (3) with,$V\to V+\alpha$.
c) Invariants: mass conservation The mass is defined by:

$$N(t)=N(\psi(t,\cdot))=\int_x|\psi|^2\,dx,\ \forall t>0. \quad (5)$$

Then we can prove that we have the mass conservation $N(t)=N(0)$.

d) Another conserved quantity is the energy: let us introduce the energy functional E_ψ as

$$E_\psi=\int_x\{\frac{1}{2}|\psi|^2+V(x)|\psi|^2+\beta(|\psi|^2)\}dx,\ \forall t>0. \quad (6)$$

where the primitive B of β is defined as:

$$B(\lambda)=\int_0^\lambda\beta(\lambda)d\lambda. \quad (7)$$

Then we have the energy conservation property,$E_\psi=E_{\psi_o}$.

e) A last property of interest is related to the dispersion relation providing hence special solutions: the plane wave

$$\psi(t,x)=ae^{i(kx-\omega t)} \quad (8)$$

is solution to system (3) (with V = 0) if the following nonlinear dispersion relation holds

$$\omega=||k||^2/2+\beta(|a|^2), \quad (9)$$

where $\|\cdot\|$ is the usual 2-norm for complex valued vector fields.
By taking all these into account the numerical solution of the equation (3) will take the form:

$$\sqrt{-1}\frac{\psi_{i,j}^*-\psi_{i,j}^n}{k}=-\frac{1}{8}\left\{\begin{matrix}[\psi_{i+1,j}^*-\psi_{i,j}^*+\psi_{i-1,j}^*+\psi_{i+1,j}^n-\psi_{i,j}^n+\psi_{i-1,j}^n]+\\ [\psi_{i,j+1}^*-\psi_{i,j}^*+\psi_{i,j-1}^*+\psi_{i,j+1}^n-\psi_{i,j}^n+\psi_{i,j-1}^n]\end{matrix}\right\}+$$

$$\frac{V_{i,j}}{2}\left\{\psi^{*}_{i,j}+\psi^{n}_{i,j}\right\}+G_{2d}\frac{\left|\psi^{n}_{i,j}\right|^{2}}{2}\left\{\psi^{*}_{i,j}+\psi^{n}_{i,j}\right\} \quad (10)$$

III. RESULT AND DISCUSSION

In this study, we examine and extend the analysis of Bose-Einstein condensation in an external harmonic potential applied along X-axis, and optical lattice potentials applied along Y-axis. The two-dimensional system of N non-interacting Bosons with the particles contained in an isotropic two-dimensional harmonic potential and optical lattice potential are analysis in this work. In this paper we have examined several features of BEC in harmonic potentials and optical lattice potential for the ideal gas theoretically which we are going to explain it as follow: First of all it's worth to mention that the working field is divide into 1000 point in space coordinates, and divide into 300000 point in time coordinates, the space interval is DX = 0.020, DY = 0.020 and the time interval is DT = 0.0001, the nonlinearity is fixed to the value of 12.548, the total number of runs for each case is 500 and the total number of pass is 3000. Under these conditions we can examine several cases along the direction of understanding of BEC as follows:

1) Figure (1) shows the relation of the chemical potential with the parameter q that appears in optical lattice potential. This parameter is responsible on the distribution of the optical lattice potential along Y-axis. The oscillator harmonic potential for this case kept constant with the value of anisotropy equal to 1.0. one can conclude from this figure that the value of the optical lattice potential play an important role in determining the chemical potential of the system. The relation of the chemical potential (μ) of the system with the optical lattice potential in term of parameter q can fit the polynomials:
   $\mu(q) = -1.432 * 10^{-8}q^6 + 4.121 * 10^{-6}q^5 - 4.177 * 10^{-4}q^4 + 0.0180q^3 - 0.291q^2 + 3.127q - 8.355.$

2) Figure (2) shows the wave function as a function of an isotropy (AL) for fixed value of optical lattice potentials. From this figure one can says that there is a logarithm relation between the wave function and the an isotropy of the harmonic oscillator potential and this relation can be written as:
   $$\psi(AL) = 0.1591\ln(AL) + 0.2103, \qquad 0.5 \le Al \le 7.0$$
   It is worth to say that this relation is restricted to the values of the anisotropy with the range $0.5 \le Al \le 7.0$

3) Figure (3) shows the chemical potential as a function of an isotropy for fixed values of optical lattice potential. This figure is true for the range of anisotropy $0.5 \le Al \le 7.0$, and the relation between the chemical potential and the an isotropy can fit a second order polynomial which is look like the one given below:
   $$\mu(AL) = 0.24994806(Al)^2 + 0.00025534148(AL) + 70.638367, \qquad 0.5 \le Al \le 7.0$$

4) Figure (4) shows a two dimension distribution of wave function on the XY-plane. This distribution is calculated for fix value of the optical lattice potential in term of the dimensionless parameter q equal to 45, and for two difference values of harmonic oscillator potential in term of the dimensionless an isotropy values which are 0.5, and 6.0 respectively. One can say from this figure that the wave function distribution of the condense gas are increases remarkable as the anisotropy increase. The transvers distribution of the wave function along the y-axis is almost controlled by the OPL only, and since the OPL is constant along this axis so, we can notice from this figure that the condense gas distribution along the transvers axis is not affected at all, although the values of the wave function are not the same.

5) In order to analysis a comprehensive view of the distribution of wave function, one have to compare between the initial and final distributions of the wave functions for a fixed values of the trapping potentials along the X-axis, and Y-axis as seen in figures (5). Figure (5a) shows the 2D distributions of wave function which is initially in the ground state of the trap potential and is therefore stationary. The effect of an optical lattice trapping potentials on this distribution is clear from this figure. The wave functionis separating into a series of localized pieces as shows in figure (5a), with the contour map of the distribution of wave function as its projected in XY plane. The potential energy operator therefore takes the form $V(x,y) = V_0\sin^2(qy) + \frac{1}{2}(AL)^2x^2$, where ALis the trap constant, qis the laser field wave number, V_0 is the lattice intensity. In applications to quantum computing, this localized wave function is to represent quantum bits. However, due to the nonlinearity of the equations, the condensate develops a phase that varies from lattice site to lattice site as seen in this figure, which is undesirable for quantum computing, since these algorithms assume that there are zero relative phases among the various single quantum bits. The problem is therefore to eliminate this phase profile by adjusting the trap strength by applying the laser field in term of the optical lattice potentials. The trap constant AL is taken as the control and the objective is to minimize the variance of the phase of the wave function. In doing that and by keeping the nonlinearity of the equations under control,

and after 500 runs with 2000 pass of the computer programs the shape of the distribution of final wave function is behave very well as expected and the phase became constant from lattice site to lattice site as one can see this clear in figure (5b). This figure shows a 2D view of the distribution of final wave function together with a contour projection of this distribution on the XY-plane. As a conclusion one can say that the optical lattice trapping potential together with the oscillator harmonic potential play an important part on the distribution of the initial and final wave functions of the condensation and by a carful adjustment of these trapping potential on can bring the calculated values to a real experiment results.

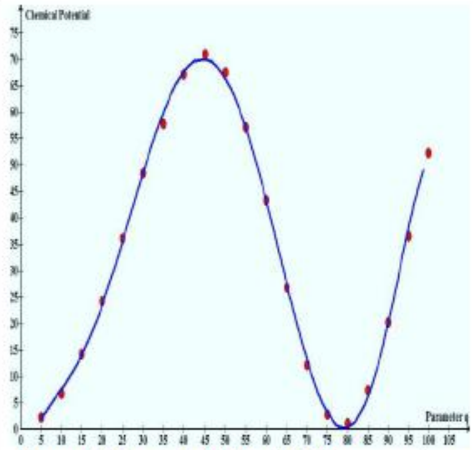

Figure (1) Chemical Potential VS parameter q AL = 1.0

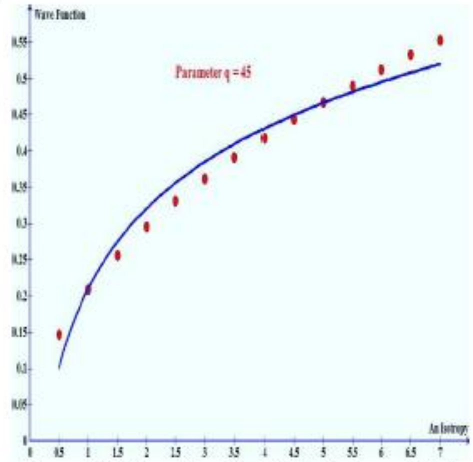

Figure (2) Wave Function VS Anisotropy Q=45

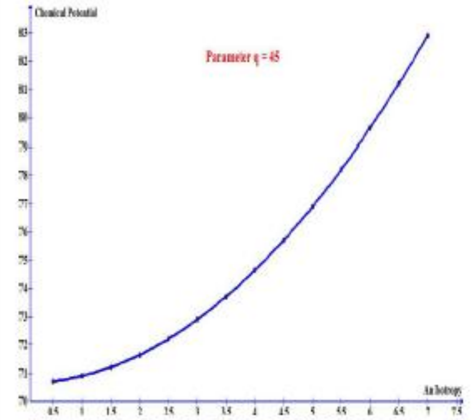

Figure (3) Chemical Potential VS AnIsotropy

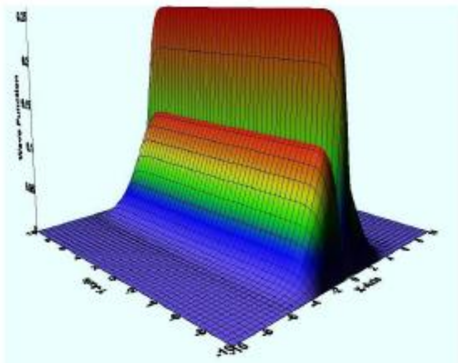

Figure (4) Distribution of the wave function on the XY-plane for fixed values of OLP at q = 45 and two difference cases for HOP AL=0.5, AL=6.0

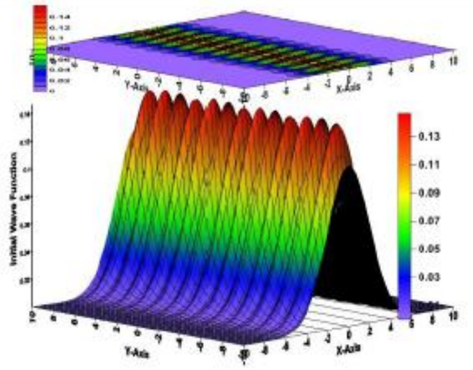

Figure (5a) Distribution of initial wave function anisotropy = 0.5 q = 45

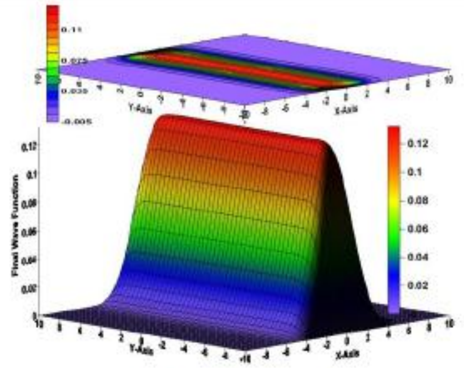

Figure (5b) Distribution of final wave functionanisotropy = 0.5 q = 45

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